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PHYSICS DEPARTMENT

E-Content

On

CONSERVATIVE AND NON-CONSERVATIVE FORCES

(Part-II)

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CONSERVATIVE AND NON-CONSERVATIVE FORCES

Summery: In this content we will discuss about conservative and non-conservative forces, some examples and their properties. Also we will find the relation between potential energy and conservative force.

Conservative force, in physics, any **force**, such as the gravitational **force** between the Earth and another mass, whose work is determined only by the final displacement of the object acted upon. ... Stored energy, or potential energy, can be **defined** only for **conservative forces**.

i.e**. If work done by a force does not depend on the path followed by object then it is known as conservative force where as if work done by the force depends on the path then it is called non-conservative force**. Or

A force on a particle will conservative when the particle come back after completing a cycle in it's initial position with the same energy which was at initial.

Suppose, an object moves from A to B in the influence of a conservative force, first, it follows path 1 and the second time it follows the path 2, work done in both conditions will be equal.

Examples of conservative forces-**Gravitational force, Elastic spring restoring force, Electrostatic force, Buoyancy force, and all central forces.**

Examples of non-conservative forces-**Friction, Air resistance, Water drag on a moving boat, Non-elastic material stress, Viscosity, Resistance to flow of electric current, and all contact forces.**

Some properties of conservative force:

If a conservative force is working on a particle then we can find,

I. The change of kinetic energy of the particle in a complete cycle is zero.
II. The work done in a complete cycle is zero.
III. The work done between two points does not depend on the path followed.
IV. Conservative force can be written as negative gradient of potential energy.

$$\vec{F} = -grad\, U.$$

V. The curl of conservative force is zero. $Curl\, \vec{F} = 0.$

Relation between conservative force and potential energy:

We have to prove that $F = -\vec{\nabla}U$.

The potential energy of a particle present in the influence of a conservative force is given by,

$$U(\vec{r}) = \int_{\infty}^{\vec{r}} \vec{F} \cdot d\vec{r}$$

The force and displacement in component form,

$$\vec{F} = F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k} \text{ } and \text{ } d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$$

Therefore,

$$\vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$

So,

$$U(\vec{r}) = -\int \vec{F} \cdot d\vec{r} = -\int F_x dx + F_y dy + F_z dz$$

$$U(\vec{r}) = -\int F_x dx - \int F_y \, dy - \int F_z dz$$

Partially differentiate with respect to x, y, z,

$$F_x = -\frac{\partial U}{\partial x}, F_y = -\frac{\partial U}{\partial y}, F_z = -\frac{\partial U}{\partial z}$$

So the force,

$$\vec{F} = F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k} = -\frac{\partial U}{\partial x}\hat{\imath} - \frac{\partial U}{\partial y}\hat{\jmath} - \frac{\partial U}{\partial z}\hat{k}$$

$\boldsymbol{\vec{F} = -\nabla\vec{U}}$ **or** $\boldsymbol{= -grad\ U.}$

Curl $\vec{F}$ = 0:

This is the condition for conservative force.

$$curl\, \vec{F} = \vec{\nabla} \times \vec{F} = \vec{\nabla} \times (-\nabla U)$$

$$= -\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$= -\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} & \frac{\partial U}{\partial z} \end{vmatrix}$$

$$= -\hat{\imath}\left(\frac{\partial^2 U}{\partial y \partial z} - \frac{\partial^2 U}{\partial z \partial y}\right) - \hat{\jmath}\left(\frac{\partial^2 U}{\partial z \partial x} - \frac{\partial^2 U}{\partial x \partial z}\right) - \hat{k}\left(\frac{\partial^2 U}{\partial x \partial y} - \frac{\partial^2 U}{\partial y \partial x}\right)$$

But ***U*** is perfect differential,

$$\frac{\partial^2 U}{\partial y \partial z} = \frac{\partial^2 U}{\partial z \partial y}$$

Curl $\vec{F}$=0.