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PHYSICS DEPARTMENT

E-Content

On

LINEAR AND ANULAR MOMENTUM AND APPLICATIONS

(Part-III)

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LINEAR AND ANULAR MOMENTUM AND APPLICATIONS

Summery:

In this content we will talk about linear and angular momentum, their conservation, examples of conservation and conservation of linear momentum of the system of two particles. In this content we will also learn about collision of two particles, types of collision, their examples and head-on collision in one dimension also we will determine the expressions for final velocities of both particles. We will discuss the collision with different conditions of mass and velocities. We will also introduced two-dimensional collision in laboratory frame.

Linear Momentum:

Linear momentum is the parameter which gives the total magnitude of motion of a body. It depends on the mass and velocity of the body. If mass and velocity of a body are $\boldsymbol{m}$ and $\vec{\boldsymbol{v}}$ respectively then momentum of that body can determine by the formula,

$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$$

Therefore, **The momentum is equal to the product of mass and acceleration.** It's direction is along the velocity and it's unit is **kg-m/sec.**

Conservation of Linear Momentum:

As per the statement, **Linear momentum of a body remains conserve if no any external force is worked**.

i.e. if $\vec{F} = 0, \vec{p} = constant.$

Let we consider a force **F** is applied on a body of mass m then according to Newton's second law,

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$

Where $\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$, $\boldsymbol{m}$ is the mass and $\vec{\boldsymbol{v}}$ is the velocity of the body.

If $\vec{F} = 0$ then,

$$\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = 0$$

Or $\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}} = \boldsymbol{constant}$.

This is the principle of linear momentum conservation. This shows that if the momentums of two bodies are equal then lighter body moves faster then the heavy body.

<u>Conservation of Linear momentum of a system of two particle:</u>

Let we consider a system of two particles and no any external force is worked on the system although a mutual force (as gravitational force)is present. Here $\vec{F}_{12}$ is the force applied by the first particle on the second particle and $\vec{F}_{21}$ is the force applied by the second particle on first particle then as per Newton's third law,

$$\vec{F}_{12} = -\vec{F}_{21}$$

Or

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

According to Newton's second law,

$$\vec{F}_{12} = \frac{d\vec{p}_1}{dt} \; and \; \vec{F}_{21} = \frac{d\vec{p}_2}{dt}$$

Where $\vec{p}_1 \; and \; \vec{p}_2$ are the momentums of first and second particle respectively.

And hence,

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$

Or

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = 0$$

$$\vec{p}_1 + \vec{p}_2 = 0$$

Therefore, in the presence of internal mutual force, the total momentum of the system of two particles remains constant. If no any external force is applied.

<u>Angular momentum:</u>

Moment of linear momentum is angular momentum.

If the position vector of a particle with respect to a fixed point is $\vec{r}$ and the linear momentum is $\vec{p}$ then angular momentum,

$$\vec{J} = \vec{r} \times \vec{p}$$

If the mass and velocity of the particle is m and $\vec{v}$ then,

$$\vec{p} = m\vec{v}$$

Therefore,

$$\vec{\boldsymbol{J}} = \vec{\boldsymbol{r}} \times (\boldsymbol{m}\vec{\boldsymbol{v}}) = \boldsymbol{m}(\vec{\boldsymbol{r}} \times \vec{\boldsymbol{v}})$$

i.e. the direction of angular momentum is perpendicular to $\vec{r}$ and $\vec{v}$ both.

Conservation of angular momentum:

If the torque on a particle is $\vec{\tau}$ and angular momentum of the particle is $\vec{J}$ then from the relation,

$$\frac{d\vec{J}}{dt} = \vec{\tau}$$

If $\vec{\tau} = 0$ then,

$$\frac{d\vec{J}}{dt} = 0$$

$$\vec{\boldsymbol{J}} = \boldsymbol{constant}$$

Examples-

I. Motion of planets and satellites.
II. Scattering of protons and alpha particles by heavy nucleus.

APPLICATION OF LINEAR MOMENTUM CONSERVATION:

Collision:

When two bodies come to each other than for a short time a force takes play between two bodies because of which the motion of bodies seems to be change and the velocities, momentums and the kinetic energy are changed. Although the momentum of both bodies changes but the total momentum of the system remains constant.

This should be understood here clearly that in Physics, it is not compulsory to come in contact of two bodies during the collision.

There are two types of collision as per the behavior of kinetic energy,

I. Elastic collision
II. Inelastic collision

Elastic collision:

Elastic collision is that collision in which the momentum and kinetic energy of the system both remain constant during (before and after) collision.

i.e. momentum conservation in elastic collision,

Total momentum before collision = total momentum after collision

$$\boldsymbol{m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2}$$

And the conservation of kinetic energy,

$$\boldsymbol{\frac{1}{2}m_1\vec{u}_1^2 + \frac{1}{2}m_2\vec{u}_2^2 = \frac{1}{2}m_1\vec{v}_1^2 + \frac{1}{2}m_2\vec{v}_2^2}$$

Where m_1, m_2 are masses, $\vec{u}_1, \vec{u}_2$ are initial velocities and $\vec{v}_1, \vec{v}_2$ are the final velocities of particles.

Examples-

I. Collision among the gases molecules.
II. Collision between nucleus and atoms.
III. Collision of the particles like electrons, protons and alpha particle.
IV. Collision between two glass balls.

Inelastic collision:

This is the collision in which momentum remains constant but the kinetic energy does not remain constant.

i.e. momentum conservation in inelastic collision,

$$\boldsymbol{m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2}$$

And the conservation of kinetic energy,

$$\boldsymbol{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2}$$

Or $\boldsymbol{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + E}$

Here $\boldsymbol{E}$ is the loss in kinetic energy in the form of sound, light and heat. This is Excitation energy.

Remember that in inelastic collision the kinetic energy of the system changes but total kinetic energy remains constant.

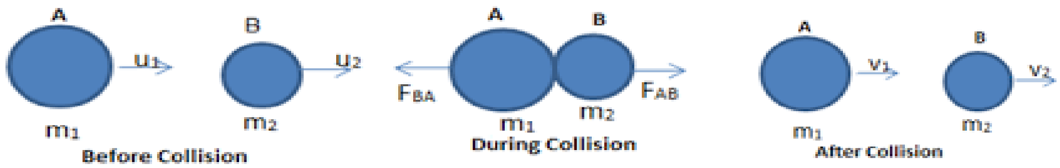
Examples-

I. Collision of meteorites with the earth.
II. Sinking of a bullet in the sack of sand etc.

ONE DIMENSIONAL OR HEAD-ON ELASTIC COLLISION

In this collision the direction of relative motion of particles remains along the same line before the collision and after collision.

Let m_1, m_2 are masses, $\vec{u}_1, \vec{u}_2$ are initial velocities and $\vec{v}_1, \vec{v}_2$ are the final velocities of particles.



Because the collision is one dimensional therefore all the velocities are in same direction. So from conservation of linear momentum,

Total momentum before collision = total momentum after collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{........ (1)}$$

Collision is inelastic so the energy also remains constant,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_1(v_2^2 - u_2^2)$$

$$m_1(u_1 + v_1)(u_1 - v_1) = m_2(v_2 + u_2)(v_2 - u_2) \quad \text{....... (2)}$$

On dividing (1) by (2),

$$(u_1 + v_1) = (v_2 + u_2)$$

$$u_1 - u_2 = -(v_1 - v_2) \quad \text{......... (3)}$$

i.e.

Relative velocity before collision = Relative velocity after collision

In one dimensional elastic collision relative velocity before collision and after collision are equal but opposite in direction.

Expression for velocities after collision:

From equation (3),

$$u_1 - v_2 = v_2 - v_1$$

$$v_2 = u_1 + v_1 - u_2$$

On putting the value of $\vec{v}_2$ in (1),

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$$

Or $-m_1v_1 - m_2v_1 = m_2u_1 - m_1u_1 - 2m_2u_2$

Or $-(m_1 + m_2)v_1 = (m_2 - m_1)u_1 - 2m_2u_2$

$$\boldsymbol{v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2}$$

Again from (3),

$$v_1 = v_2 + u_2 - u_1$$

On putting the value of v_1 in (1),

$$m_1(u_1 - v_2 - u_2 + u_1) = m_2(v_2 - u_2)$$

or $m_1(2u_1 - v_2 - u_2) = m_2(v_2 - u_2)$

or $-m_1v_2 - m_2v_2 = -2m_1u_1 + m_1u_2 - m_2u_2$

or $-(m_1 + m_2)v_2 = -2m_1u_1 + (m_1 - m_2)u_2$

$$\boldsymbol{v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_2}$$

Some special condition:

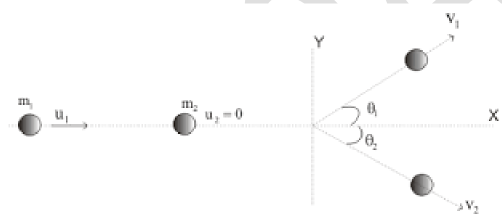
1. If the masses of both particles are equal i.e. $m_1 = m_2$ then $v_1 = u_2\ and\ v_2 = u_1$ i.e. the velocities of particles will exchange.
2. If the particle of mass m_2 is in rest i.e. $u_2 = 0$ then $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1\ and\ v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1$ and also $m_1 = m_2$ then $v_1 = 0\ and\ v_2 = u_1$ i.e. the second particle moves with the velocity of first particle and the first particle becomes in rest.

3. If $u_2 = 0$ and $m_1 \ll m_2$ then $v_1 \approx -u_1$ *and* $v_2 = 0$ i.e. the light particle moves with same velocity where as the heavy particle remains in rest.
4. If $u_2 = 0$ and $m_1 \gg m_2$ then $v_1 \approx u_1$ *and* $v_2 = 2u_1$ i.e. the velocity of heavy particle not changes where as the light particle moves with the velocity twice of the velocity of heavy particle.

TWO DIMENSIONAL ELASTIC COLLISION:

We will study two-dimensional collision in laboratory frame. We will determine expressions for v_1 *and* v_2 and also find the condition for the direction of both particles after collision.

Let a particle of mass m_1 and velocity u_1 collides elastically with the other particle of mass m_2 which is in rest. After collision the incident particle moves with the velocity v_1 making an angle θ_1 with it's initial direction and the target particle moves with the velocity v_2 making an angle θ_2 with the direction of first particle. The angle θ_1 is called scattering angle



and θ_2 is called recoil angle.

From the conservation of linear momentum,

In X-direction,

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{..... (1)}$$

In Y-direction,

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \quad \text{.... (2)}$$

Because the collision inelastic the kinetic energy also remains constant,

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \text{.... (3)}$$

Let θ_1 is known and $m_1 = m_2 = m$ then (1), (2) and (3) will be,

$$u_1 = v_1 \cos\theta_1 + v_2 \cos\theta_2 \quad \text{.... (4)}$$

$$0 = v_1 \sin\theta_1 - v_2 \sin\theta_2 \quad \text{.... (5)}$$

$$\boldsymbol{u_1^2 = v_1^2 + v_2^2} \quad \text{.... (6)}$$

Or

$$u_1 - v_1 \cos\theta_1 = v_2 \cos\theta_2 \quad \text{.... (7)}$$

$$v_1 \sin \theta_1 = v_2 \sin \theta_2 \qquad \text{.... (8)}$$

$$\boldsymbol{u_1^2 - v_1^2 = v_2^2} \qquad \text{.... (9)}$$

On squaring and adding (7) and (8),

$$(u_1 - v_1 \cos \theta_1)^2 + (v_1 \sin \theta_1)^2 = (v_2 \cos \theta_2)^2 + (v_2 \sin \theta_2)^2$$

$$u_1^2 + v_1^2 - 2u_1 v_1 \cos \theta_1 = v_2^2 \qquad \text{.... (10)}$$

On subtracting (9) from (10),

$$2v_1^2 - 2u_1 v_1 \cos \theta_1 = 0$$

$$2v_1^2 = 2u_1 v_1 \cos \theta_1$$

$$v_1 = u_1 \cos \theta_1 \qquad \text{.... (11)}$$

Equation (11) gives the value of v_1 in terms of θ_1 *and* u_1.

On putting the value of v_1 in (9),

$$u_1^2 - u_1^2 cos^2\theta_1 = v_2^2$$

$$u_1^2(1 - cos^2\theta_{1)} = v_2^2$$

$$u_1^2 sin^2\theta_1 = v_2^2$$

$$v_2 = u_1 \sin \theta_1 \qquad \text{.... (12)}$$

This equation gives the value of v_2 in terms of u_1 *and* θ_1.

On dividing (8) by (7),

$$\tan \theta_2 = \frac{v_1 \sin \theta_1}{u_1 - v_1 \cos \theta_1}$$

On putting the value of v_1 from (11),

$$\tan \theta_2 = \frac{u_1 cos\theta_1 sin\theta_1}{u_1 - u_1 cos^2\theta_1}$$

$$= \frac{u_1 cos\theta_1 sin\theta_1}{u_1(1 - cos^2\theta_1)}$$

$$= \frac{u_1 cos\theta_1 sin\theta_1}{u_1 sin^2\theta_1}$$

$$\frac{\cos\theta_1}{\sin\theta_1} = \cot\theta_1$$

$$= \tan\left(\frac{\pi}{2} - \theta_1\right)$$

Therefore,

$$\boldsymbol{\theta_2 = \frac{\pi}{2} - \theta_1} \qquad \dots.(13)$$

This equation gives the value of θ_2 in terms of θ_1.

From (13),

$$\boldsymbol{\theta_1 + \theta_2 = \frac{\pi}{2}}$$

This shows that the particles move perpendicular to each other.