

Website: [www.drbagdcmainpuri](http://www.drbagdcmainpuri)

Email: [singh525sps@yahoo.com](mailto:singh525sps@yahoo.com)

**Dr. B.R. Ambedkar Government Degree College**  
Mainpuri (UP)~ 205001

डॉ० बी०आर० अम्बेडकर राजकीय महाविद्यालय, ओडन्य पडरिया, मैनपुरी (उ०प्र०)

**Name: Dr S. P. Singh**

Assist. Professor, Physics Department

Mobile No: +919411205673



Session: 2020-21

**PHYSICS DEPARTMENT**

---

**E-Content**

**On**

**Laws of Thermodynamics**

**(Part-I)**

**Author: Dr. Surendra Pratap Singh**

**Affiliation: Dr. B. R. Ambedkar Government Degree College, Mainpuri (UP)-205001**

**(Affiliated to Dr. Bhimrao Ambedkar University, Agra)**

## Laws of Thermodynamics

**Basic Objectives:** The Students will be introduced basic concepts and definitions of thermodynamic related parameters, properties and Laws. This topic divided in two parts.

**First part** contains the basic definition of thermodynamic parameters, processes and systems, equation of states, concepts of internal energy, first law of thermodynamics.

Reversible and irreversible changes, Carnot cycle and its efficiency, Carnot theorem, Second law of thermodynamics. Entropy, principle of increase of entropy and calculations.

The thermodynamic scale of temperature and its identity with the perfect gas scale.

**Second part** contains the thermodynamic relations, thermodynamics variables, deduction of Maxwell's relations, Thermodynamic potentials and equilibrium of thermodynamical systems, relation with thermodynamical variables.

Application to Joule-Thomson cooling and adiabatic cooling in a general system,

Clausius-Clapeyron heat equation.

**1. Thermodynamics:** Thermodynamics is the branch of the physics, which represent the transformations of the Heat into the Mechanical Work or Mechanical work transforms into the heat.

Mathematically, Let us assume that heat  $Q$  produce during work done  $W$ ,

$$W = QJ \quad \text{----- (1.1)}$$

Where  $J$  is universal constant known as Mechanical heat constant and its value is  $J = 4.18 \times 10^7$  Erg/cal.

**Question No. 1:** Waterfalls from height of 84 m on the ground. Half of energy of water is converted into heat. What is rise in temperature of water when touches ground?

**Solution:** Here work done = change in potential energy

$$= mgh \text{ Joule}$$

$$\text{Heat Produced } Q = \frac{W}{J}$$

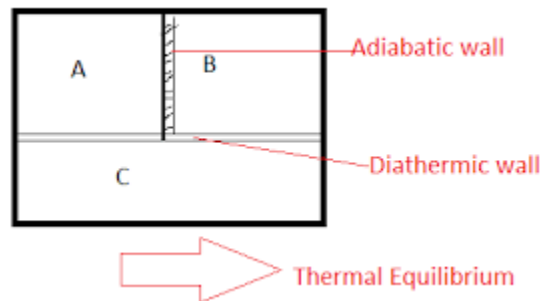
$$\Rightarrow Q = \frac{\frac{1}{2}mgh}{J}$$

$$\Rightarrow ms\Delta T = \frac{mgh}{2J}$$

$$\Rightarrow \Delta T = \frac{mgh}{2sJ} = \frac{9.8 \times 84}{2 \times 4.2 \times 10^3 \times 1} = 0.098 \text{ } ^\circ\text{C}$$

**1.1. Thermodynamical Variables or Thermodynamical Co-ordinates or Thermodynamical parameters:** Thermodynamic state of the system represents in terms of their macroscopic properties i.e. pressure, temperature, volume entropy, enthalpy etc. These macroscopic properties of the system are called *thermodynamic co-ordinates or thermodynamic variables or thermodynamic parameters*. These quantities are the function of gas states so that it is also known as *State Functions or States variables*.

**1.2 Zeroth Law of thermodynamics:** The *Zeroth Law of Thermodynamics* states that when two systems are in the thermal equilibrium with the third system then they must be in the thermal equilibrium with each other.



**Fig.1**

**1.3 Zeroth Law and temperature:** The temperature of the system as the property that determines whether or not the system is in thermal equilibrium with the neighbor systems.

**1.4 Equation of States:** In order to describe the thermodynamic state of a system therefore some certain variables are required. These variables are generally the mechanical variables augmented by the thermal state of the system. Thus the equation is connection or functional relationship of the pressure, the volume and the absolute temperature of a matter such equation known as Equation of States (EoS). Thus general equation given as follows:

$$f(P, V, T) = 0 \quad \text{----- (1.2)}$$

**1.5 General Equation:** This eq. (1.2) can be solving with respect to any variable. The general solution of the eq. (1.2) given as

$$P \equiv P(V, T) \quad \text{----- (1.3)}$$

$$V \equiv V(P, T) \quad \text{----- (1.4)}$$

$$T \equiv T(P, V) \quad \text{----- (1.5)}$$

**1.5.1 Perfect Gas or Ideal Gas Equation (PVT-Relation):**

The perfect gas equation for one gram mole is given as

$$PV = RT \quad \text{----- (1.6)}$$

For n gram mole is given as

$$PV = nRT \quad \text{----- (1.7)}$$

Where P stand for pressure of gas, V be the volume and T be the reference temperature of gas.

R is the gas constant it is universal. The numerical value and its unit of R given as

$$R = Nk = 8.31 \frac{J}{Mol - K}$$

$$R = 8.31 \frac{J}{kg Mol - K}$$

Where  $N = Avogadro\ Number = 6 \times 10^{23}$  per gm mole

= Number of Molecules in 1 gram mole of a gas.

$k = Boltzmann's\ constant$

$$= 1.38 \times 10^{-23} J/K$$

### 1.5.2 Real Gas or Vander Waal's Gas Equation:

Vander Waal's gas equation for one gram mole is given as

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \quad \text{----- (1.8)}$$

For n gram mole

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT \quad \text{----- (1.9)}$$

Where a and b are Vander Waal's gas constants.

**Unit of a:**

$$a = \frac{Atoms\ Litre^2}{Mole^2}$$

$$a = 10^{12} \frac{Dyne\ cm^4}{Mole^2}$$

**Unit of b:**

$$b = \frac{Litre}{Mole} = \frac{cc}{Mole} = \frac{Met^3}{Mole}$$

**1.6 Internal Energy:** The internal energy may be defined as the energy possessed by a body or by a system due to molecular motion and molecular configuration. The energy due to molecular motion is called internal kinetic energy ( $U_k$ ) and that due to molecular configuration is called the internal potential energy ( $U_p$ ). Thus

$$U = U_k + U_p$$

#### 1.6.1 The Physical Interpretation of Change in Internal Energy:

The physical interpretation of change in internal energy  $dU = U_f - U_i$  is the increase in the energy of the system. Thus there exists an energy function the difference in two values of which is equal to the work done in change the system i to f. Thus

- Internal energy is a function of state.

- Internal energy depends upon the initial and final state i.e. the internal energy is path independent function.  $\Delta U = U_f - U_i$

Hence it is called unique functions but work done by gas depends upon the path.

- Internal energy depends upon the temperature.
- In a cyclic process or closed system, the internal energy is zero ( $\Delta U = 0$ ).
- Internal energy of a perfect gas or ideal gas depends only on the temperature and not on its pressure and volume. It is also known as Joule's Law.

$$U = U_k = \frac{3}{2}nRT$$

- Internal energy of real gas or Vander Waal's gas, there is a force of attraction between the molecules of gas. Hence their potential energy is not zero. Therefore internal energy of a real gas depends on all pressure (P), Volume (V) and Temperature (T).

**1.7 Temperature:** Although temperature is a familiar property, an exact definition of it is difficult. Thus, we define equality of temperatures. Two bodies have equality of temperatures if, when they are in thermal communication, no change in observable property occurs.

**1.8 Pressure:** Pressure is defined as the normal component of force per unit area. Most thermodynamics investigations are concerned with absolute pressure.

**1.9 Concept of Heat:** According to the thermodynamically, *the flow of energy from one body to another an account of difference of temperature between them is called Heat*. Thus the flow of heat is a non-mechanical energy transfer brought about by a temperature difference between two bodies.

Some important silent features regarding the Heat

- Heat is path dependent function.
- It is taken positive if the system absorbs heat and negative if release it.
- In case of solids and liquids

$$\Delta Q = mL; \text{ When state is change of the matter.}$$

$$\Delta Q = ms\Delta T; \text{ When temperature is change by } \Delta T.$$

- For gases, the heat is absorbed and temperature changes for n gm mole of gas

$$(\Delta Q)_V = nC_V \cdot \Delta T; \text{ When volume is constant.}$$

$$(\Delta Q)_P = nC_P \cdot \Delta T; \text{ When pressure is constant.}$$

**1.10 First Law of thermodynamics:** The *First Law of Thermodynamics* is general the form of *Law of Conservation of Energy*. It can be represent as follow, thus “*When mechanical work is spent in producing heat, a definite quantity of heat is produced for unit of work spent; and vice versa.*” Thus the amount of heat Q produced by spent of the amount work done by a system and given as

$$W = JQ$$

$$\begin{aligned} \text{Where } J &= 4.2 \times 10^7 \text{ Erg/cal} \\ &= 4.2 \text{ Joule/cal} \end{aligned}$$

Mathematically it can be express as

Let us consider any system reach from the initial state i to final sate f using different path, then change in internal energy of system

$$dU = U_f - U_i = Q - W \quad \text{----- (1.10)}$$

Where  $U_i$  = Internal energy in initial state

$U_f$  = Internal energy in final state

$W$  = Work done by the system

$Q$  = Absorb heat by the system

The Eq. (1.10) is the Mathematical form of First Law of Thermodynamics.

The Differential form of First Law of Thermodynamics can be express as follow

If dQ heat given to a system in any process, work done by system be the dW and change in internal energy be dU, then

$$dQ = dU + dW \quad \text{----- (1.11)}$$

i.e. Heat given to the system = Change in internal energy + Work done by the system

This eq. (1.11) is the differential form of first law of thermodynamics. The Unit of dW, dU, and dQ should be same form.

**1.10.1 Physical Significance of first law of thermodynamics:** In the first law of thermodynamics tell us four silent facts:

1. Heat is a form of energy.
2. The total energy of the system remains conserved.
3. The Energy of the system in equilibrium possesses an internal energy which is a state function. It means the internal energy depends only upon initial and final state of the system.
4. The first law of thermodynamics does not us about the how many part of energy convert in heat and vice versa.

**Question No.2:** Fill up the table given below all quantities are taken in Joules. Which process is adiabatic?

Process	Q	W	$U_i$	$U_f$	$dU = U_f - U_i$
1	25	-10	---	-10	---
2	-10	---	---	65	-20
3	---	-20	80	100	20

**Solution:**

(i)  $dU = U_f - U_i = Q - W = 25 - (-10) = 35 \text{ Joule}$

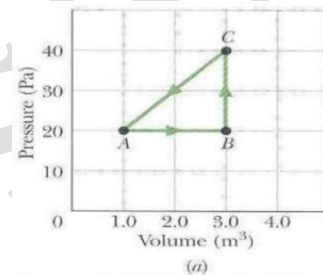
(ii)  $dU = -10 - W = -20 \Rightarrow W = 10 \text{ Joule}$

(iii)  $dU = Q - W = Q - (-20) = 100 - 80 \Rightarrow Q = 0 \text{ Joule}$

Because of the third (3) process Q becomes zero so that the third (3) process in the table will be adiabatic.

**Question No.3:** A thermodynamic system is taken from the state A to B and then state C as shown on PV-diagram below. (i) Write in table +ve and -ve (ii) find work done in ABCA.

Process	dQ	dW	dU
A → B		+ve	+ve
B → C			+ve
C → A			-ve



**Solution: (i) a. For Process (A → B)**  $dW_{A \rightarrow B} = PdV = 20(3 - 1) = 40 = +ve$

**b. For Process (B → C)**  $dW_{B \rightarrow C} = PdV = 20(3 - 3) = 0 = +ve$

**c. For Process (C → A)** The gas is compressed. The volume is decrease and work done  $dW_{C \rightarrow A} = -ve$

(ii) The work done for ABCA = Area of Triangle ABCA

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times (3 - 1) \times (40 - 20)$$

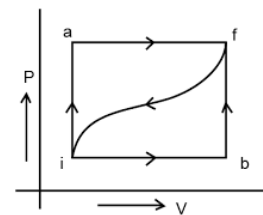
$$= 20 \text{ Joule}$$

**Question No.4:** When a system is taken from state i to f along the path iaf we find  $dQ = 50 \text{ cal.}$ ,  $dW = 20 \text{ Cal.}$  While along the path ibf  $dQ = 36 \text{ Cal.}$  If  $U_i = 10 \text{ Cal.}$  Find

(i) What is  $dW$  along ibf ?

(ii) if  $dW = 13 \text{ cal}$  along the curved path fi what  $dQ$  this path?

(iii) if  $U_b = 22 \text{ Cal.}$  What  $dQ$  for the path ib and bf?



**Fig. (a)**

**Solution:**

(i)  $dW_{ibf} = ?$

Given  $dQ = 36 \text{ Joule}$

$$\text{As } dQ = dU + dW$$

$$\Rightarrow 36 = dU + dW_{ibf} \text{ or } 36 = 30 + dW_{ibf} \text{ or } dW_{ibf} = 36 - 30 = 6 \text{ Cal.}$$

$$\Rightarrow dQ = dU + dW_{i \rightarrow f}$$

$$\Rightarrow 50 = dU + 20$$

$$\Rightarrow dU = 30 = U_f - U_i$$

$$\Rightarrow U_f = 30 + 10 = 40 \text{ Cal.}$$

$$\text{(ii) } dU_{if} = U_f - U_i = -(U_i - U_f) = -30 \text{ Cal.}$$

$$\therefore dQ = dU + dW = -30 - 13 = -43 \text{ Joule}$$

$$\text{(iii) } dW_{bf} = 0 \text{ and } dU_{bf} = U_f - U_b = 40 - 22 = 18 \text{ Cal.}$$

$$dU_{bf} = dQ_{bf} = 18 \text{ Cal.}$$

$$U_{ib} = U_b - U_i = 22 - 10 = 12 \text{ Cal.}$$

$$dQ_{ib} = dU_{ib} + dW_{ib} = 6 + 12 = 18 \text{ Cal.}$$

**1.11 Thermodynamic Equilibrium:** When the conditions for all three types of equilibrium, thermal, mechanical, and chemical, are satisfied, the system is said to be in thermodynamic equilibrium. However, if the conditions for any one of three types of equilibrium are not satisfied, the system will be in a thermodynamic non equilibrium.

**1.11.1 Thermodynamic Process:** Any change which causes the thermodynamic variables of a system to have a new set of values is called the *thermodynamic process*.

**1.11.2 Quasistatic Process:** If the change in the thermodynamic system has been done so slowly, then the state of system remains arbitrarily close to the equilibrium position, thus this change is called the *Quasistatic process*. The Quasistatic Process can be represented on the *PV-diagram* or *Indicator diagram*. The necessary feature for the *Quasistatic process is slowness*.

These processes are four types.

(a) Isothermal Process:  $\Delta T = 0 \Rightarrow T = \text{Constant}$  (The Temperature remains constant of the system.)

(b) Adiabatic Process:  $\Delta Q = 0 \Rightarrow Q = \text{Constant}$  (The Heat remains constant of the system.)

(c) Isobaric Process:  $\Delta P = 0 \Rightarrow P = \text{Constant}$  (The Pressure remains constant of the system.)

(d) Isochoric Process:  $\Delta V = 0 \Rightarrow V = \text{Constant}$  (The Volume remains constant of the system.)

**1.11.2 (a) Isothermal Process:** When such a change in any thermodynamic system in such a way that its *temperature* remains constant, then it is called the *Isothermal Process*.

In such a process, the heat produced or heat lost is given out to or taken from the surrounding in such a way that the temperature of the system remains constant. This process should be going very slowly. **Ex-** *Melting of ice at its melting point.*



**Work done in Isothermal Process:** Let  $T^{\circ}\text{K}$  be the temperature of one gm mole of perfect gas and gas expand its initial volume  $V_1$  to final volume  $V_2$ .

The External work done by the gas =  $W = \int P dV$

Where  $P$  is the instant pressure of the gas.

For isothermal process, the perfect gas equation takes the form

$$PV = \text{Constant} = K.$$

Hence  $P = \frac{K}{V}$ , then

The External work done by the gas =  $W = \int_{V_1}^{V_2} \frac{K}{V} dV = K \log_e \left( \frac{V_2}{V_1} \right)$

$$\text{But } P_1V_1 = P_2V_2 = P_3V_3 = \dots = \text{Constant} = K \quad \text{--- (1.12)}$$

Hence

$$W = P_1V_1 \log_e \left( \frac{V_2}{V_1} \right) = P_2V_2 \log_e \left( \frac{V_2}{V_1} \right) = RT \log_e \left( \frac{V_2}{V_1} \right) \quad \text{--- (1.13)}$$

**Case I:** when the gas will expand ( $V_2 > V_1$ ), then work done  $W = +ve$  i.e. Work done by the gas.

**Case II:** when the gas will expand ( $V_2 < V_1$ ), then work done  $W = -ve$  i.e. Work done on the gas.

**1.11.2 (b) Adiabatic Process:** When such a change in any thermodynamic system in such a way that its *heat* remains constant, then it is called the **Adiabatic Process**. In such process no heat is allowed either to enter the system or to leave it from the surroundings, but along the process, there is a change in temperature. Thus all processes that take place quickly or suddenly are adiabatic processes.

**Ex-** The sudden expansion of the enclosed air into the atmosphere when a motor tyre bursts is adiabatic and hence the tyre is cooled.

**Equation of States for adiabatic process:**

**Case-I (Relation between the Pressure and the Volume):** Let us consider the temperature ( $T$ ), the Volume ( $V$ ) and Pressure ( $P$ ) of one gram mole of perfect gas. If the adiabatic change is being on in the gas, then from the First Law of Thermodynamics,

$$dQ = dU + dW$$

But for adiabatic process  $dQ = 0$

$$\text{Thus,} \quad 0 = dU + dW \text{ or } dU = -dW \text{ or } -dU = dW \quad \text{--- (1.14)}$$

The gas will spend its internal energy to do the external work done. Thus the temperature of the gas will fall due to lack of its internal energy. Therefore

$$dU = C_v dT \quad \text{--- (1.15)}$$

From eq. (1.14) and eq. (1.15), we have

$$dU = -dW = C_V dT \quad \text{--- (1.16)}$$

Where  $C_V =$  The specific heat at constant volume, and  $dW = PdV$

$$-PdV = C_V dT \quad \text{--- (1.17)}$$

The perfect gas equation for one gram mole,  $PV = RT$  --- (1.18)

By differentiating this eq. (1.18), we have

$$PdV + VdP = RdT \Rightarrow dT = \frac{PdV + VdP}{R} \quad \text{--- (1.19)}$$

By using eq. (1.17) and eq. (1.19), therefore

$$-PdV = \frac{PdV + VdP}{R} \quad \text{--- (1.20)}$$

According to Mayer's Formula

$$C_P - C_V = R \quad \text{--- (1.21)}$$

Using eq. (1.20) and eq. (1.21), we have

$$\begin{aligned} -C_P PdV &= C_V VdP \\ \frac{-C_P}{C_V} \frac{dV}{V} &= \frac{dP}{P} \end{aligned} \quad \text{--- (1.22)}$$

But the specific heat ratio is given as follow

$$\frac{C_P}{C_V} = \gamma \quad \text{--- (1.23)}$$

Thus the eq. (1.22) can be written as

$$-\gamma \frac{dV}{V} = \frac{dP}{P} \quad \text{--- (1.24)}$$

On integrating the eq. (1.24), we have

$$-\gamma \log_e V = \log_e P + \text{Constant}$$

Thus,  $PV^\gamma = \text{Constant} = K$  --- (1.25)

This is called the **Poisson's Equation in terms of the Pressure and the Volume.**

Generalizing the eq. (1.25) as thus,

$$P_1 V_1^\gamma = P_2 V_2^\gamma = P_3 V_3^\gamma = \dots = \text{Constant} = K \quad \text{--- (1.26)}$$

**Case-II (Relation between the temperature and the Volume):**

We know that  $PV^\gamma = K$  as  $PV = RT \Rightarrow P = \frac{RT}{V}$ , then

$$\begin{aligned} \Rightarrow \frac{RT}{V} V^\gamma &= K \\ \Rightarrow TV^{\gamma-1} &= K' \end{aligned} \quad \text{--- (1.27)}$$

Generalizing the eq. (1.27) as thus,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} = T_3 V_3^{\gamma-1} = \dots = \text{Constant} = K' \quad \text{--- (1.28)}$$

This is the **Poisson's Equation in terms of the temperature and the Volume.**

**Case-III (Relation between the Pressure and the Temperature):**

We know that  $PV^\gamma = K$  as  $PV = RT \Rightarrow V = \frac{RT}{P}$ , then

$$\Rightarrow P \left( \frac{RT}{P} \right)^\gamma = K$$

$$\Rightarrow P^{1-\gamma} T^\gamma = K'' \quad \text{--- (1.29)}$$

$$\Rightarrow P^{1-\gamma/\gamma} T = K'' \quad \text{--- (1.30)}$$

Generalizing the eq. (1.29) as thus,

$$P_1^{1-\gamma} T_1^\gamma = P_2^{1-\gamma} T_2^\gamma = P_3^{1-\gamma} T_3^\gamma = \dots = \text{Constant} = K''$$

This is the **Poisson's Equation in terms of the temperature and the Pressure.**

**Work done in adiabatic Process:** Let  $T_1$  be the temperature,  $V_1$  the volume, and  $P_1$  the pressure of one gm mole of perfect gas at initially, and due to adiabatic change in the gas the temperature  $T_2$ , the Volume  $V_2$ , and the Pressure  $P_2$  becomes after. Thus

$$\text{The work done} = W = \int P dV$$

For the adiabatic process the equation of state denoted as  $PV^\gamma = K$

$$\text{Thus, work done} = W = \int_{V_1}^{V_2} \frac{K}{V^\gamma} dV \quad \therefore P_1 V_1^\gamma = P_2 V_2^\gamma = P_3 V_3^\gamma = \dots = K$$

$$W = \frac{1}{1-\gamma} (P_2 V_2 - P_1 V_1) \quad \text{--- (1.31)}$$

$$W = \frac{R}{1-\gamma} (T_2 - T_1) \quad \therefore P_1 V_1 = RT_1 \text{ \& } P_2 V_2 = RT_2$$

$$W = \frac{R}{\gamma-1} (T_1 - T_2) \quad \text{--- (1.32)}$$

Because  $\frac{R}{\gamma-1}$  is a constant so that *the work done depends upon the initial and final temperatures  $T_1$  and  $T_2$ . In the other words the work done along any adiabatic between two isothermals is independent of the particular adiabatic.*

**Case I:** if  $W = \frac{R}{\gamma-1} (T_1 - T_2) = +ve$ , then  $T_1 - T_2$  should be  $+ve$  ( $T_1 > T_2$ ); *so, the gas will be cool down.*

**Case II:** if  $W = \frac{R}{\gamma-1} (T_1 - T_2) = -ve$ , then  $T_1 - T_2$  should be  $-ve$  ( $T_1 < T_2$ ); *so, the gas will be heated.*

**1.11.2 (b) Isobaric Process:** When such a change in any thermodynamic system in such a way that its *pressure* remains constant, then it is called the **Isobaric Process**. Therefore

$$\text{Change in Pressure } \Delta P = 0$$

$$\therefore P = \text{Constant} \text{ (The Pressure remains constant of this system.)}$$

**1.11.2 (c) Isochoric Process:** When such a change in any thermodynamic system in such a way that its *volume* remains constant, then it is called the **Isochoric Process**. Therefore

$$\text{Change in Volume } \Delta V = 0$$

$\therefore V = \text{Constant}$  (The Volume remains constant of this system.)

**1.11.2 (d) Cyclic Process:** In the cyclic process, the system passing from initial state to final state come back to its initial state. So, in this process the net change in the internal energy of the system is zero.

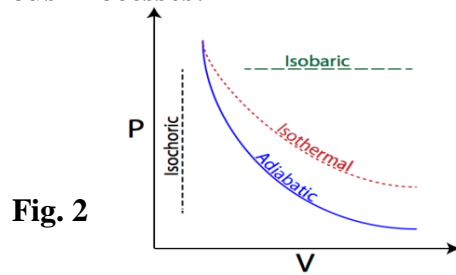
Hence from First Law of Thermodynamic  $\Delta U = Q - W$

We have  $\Delta U = 0$  so,  $Q - W = 0$  or  $Q = W$

It means, in the cyclic process the heat taken by the system is equal to the work done by the system. Thus for cyclic process the differential form of First Law of Thermodynamic will takes the form as follow  $\oint dQ = \oint dW$  ( $\therefore \oint dU = 0$ )

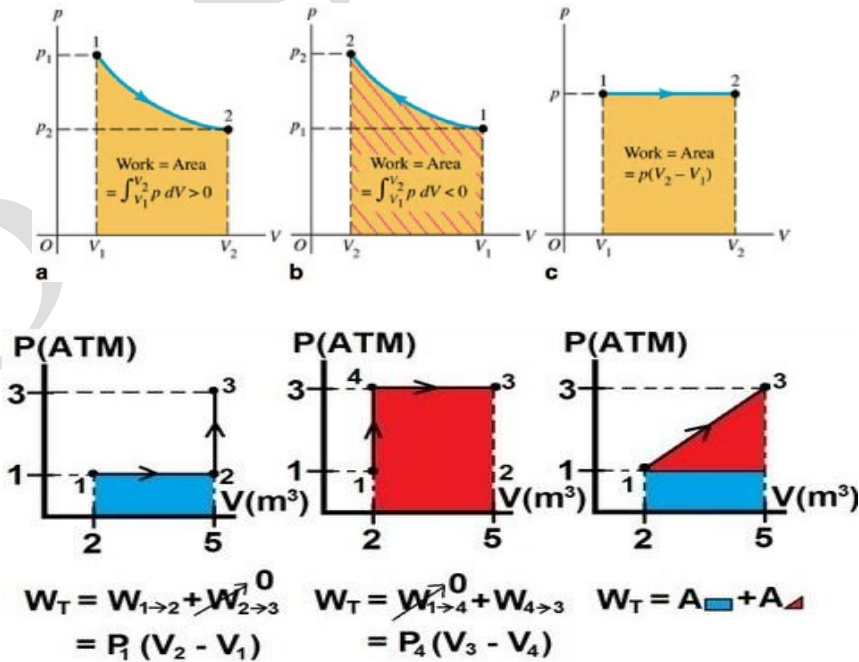
Thus, In a cyclic process, the heat given to the thermodynamic system is equivalent to the net work done by the system.

**PV -Diagram of Various Processes:**

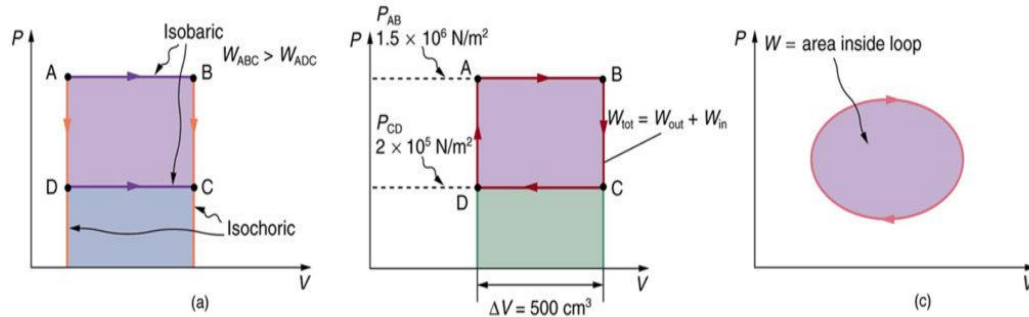


**Note: 1.** The Area covered by any process from initial to final state represents the work done by the system.

**2.** The Area covered in any cyclic process is represents the work done by the system.



**Fig.3: For the various processes.**



**Fig.4: For the various cyclic processes.**

**Question No. 5:** Prove that adiabatic is steeper than isothermal.

Or

Show that at any point the slope at adiabatic curve of a perfect gas is  $\gamma$  times of the slope of isothermal curve to through the same point.

**Solution:** The perfect gas equation for one gram mole

$$PV = RT$$

For the isothermal process

$$\Delta T = 0 \Rightarrow T = \text{constant i.e. } PV = \text{constant} \quad \text{----- (a)}$$

By differentiating eq. (a), we have

$$PdV + VdP = 0 \Rightarrow \left(\frac{dP}{dV}\right)_{iso} = -\frac{P}{V} \quad \text{----- (b)}$$

This is the slope of the isothermal curve.

But, for the Adiabatic Process;

$$PV^\gamma = \text{Constant} \quad \text{----- (c)}$$

Now, differentiating the eq. (c), we have

$$\gamma PV^{\gamma-1} dV + V^\gamma dP = 0 \Rightarrow \left(\frac{dP}{dV}\right)_{adi} = -\gamma \frac{P}{V} \quad \text{----- (d)}$$

This is the slope of the adiabatic curve.

Now using the eq. (b) and eq. (d), therefore

$$\frac{\left(\frac{dP}{dV}\right)_{adi}}{\left(\frac{dP}{dV}\right)_{iso}} = -\frac{\gamma \frac{P}{V}}{-\frac{P}{V}} = \gamma > 1$$

Hence,

$$\left(\frac{dP}{dV}\right)_{adi} > \left(\frac{dP}{dV}\right)_{iso}$$

i.e. The slope of the adiabatic curve is greater than the slope of the isothermal curve.

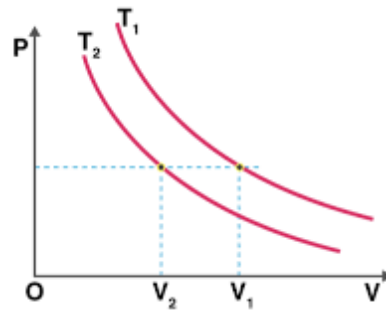
In the other words, the slope of the adiabatic curve is steeper than the slope of the isothermal curve.

**Question No. 6:** Prove that two isotherms cannot intercept each other.

**Solution:** Let two isotherm having the temperatures  $T_1$  and  $T_2$ . Those Equations are given as follow:

$$P_1V_1 = RT_1 \quad \text{----- (a)}$$

$$P_2V_2 = RT_2 \quad \text{----- (b)}$$



**Fig.5**

If these two isotherms cut the intercept to each other at any point, then  $T_1 = T_2 = T$  (say) at the cutting points. Hence,  $P_1V_1 = P_2V_2 = RT_1 = RT_2 = RT$  (which is not possible)

It is clear that two isotherms cannot intercepts to each other.