Website: www.drbagdcmainpuri Email: singh525sps@yahoo.com

Dr. B.R. Ambedkar Government Degree College
Mainpuri (UP)~ 205001
डॉ0 बी0आर0 अम्बेडकर राजकीय महाविद्यालय, ओडन्य पडरिया, मैनपुरी (उ0प्र0)

Name: Dr S. P. Singh
Assist. Professor, Physics Department
Mobile No: +919411205673

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PHYSICS DEPARTMENT

E-Content

On

Second Law of thermodynamic and its application.

(Part-II)

Author: Dr. Surendra Pratap Singh

Affiliation: Dr. B. R. Ambedkar Government Degree College, Mainpuri (UP)-205001

(Affiliated to Dr. Bhimrao Ambedkar University, Agra)

<u>**The Second Law of Thermodynamics and its Applications.**</u>

Basic Objectives: The Students will be introduced basic concepts of second law of thermodynamics, The Carnot's cycle, Efficiency of Carnot's engine, Carnot's theorem and The thermodynamic scale of temperature and its identity with the perfect gas scale.

This part contains

- The Second law of thermodynamics
- The Carnot cycle and its efficiency, Carnot theorem,
- The thermodynamic scale of temperature and its identity with the perfect gas scale.

1. Second Law of Thermodynamics: The First Law of Thermodynamics is the form of Law of Conservation of Energy. This Law states that the equivalence of the heat and the energy. But this law does not tell about some facts:

(i) How many part of Heat convert in the work?

(ii) The condition necessary for such conservation?

It was the quest for replies of several such questions which led to the formulation of Second Law of Thermodynamics. The Second Law of Thermodynamics has been stated in a number of ways, but all the statements, through different words, are logically equivalent to one another.

"Heat cannot flow of itself from cold body to the hot body."

1.1 Kelvin - Planck Statement of the Second Law of Thermodynamics: According to Kelvin - Planck Statement of the Second Law of Thermodynamics given as follows:

"It is impossible to construct a device that operating in a complete cycle will produce no other effect than the transfer of heat from a cold body to a hot body."

1.2 Clausius Statement of the Second Law of Thermodynamics: According to him the Second Law of Thermodynamics given as follows:

"It is impossible for a self acting machine working in a cyclic process, unaided by any external agency to transfer heat from a cold body to a hot body."

2. Carnot's Engine

2.1 Heat Engine: *A heat engine is a device that continuously converts the heat into mechanical work.* In the other words *Heat engine is the device which converts the chemical energy of fuel in to heat energy and this heat energy is utilized converting into Mechanical work.*

The heat engine has to have four.

1. Source of heat (T_1 ^{0}K)
2. Sink (T_2 ^{0}K)
3. Working Substance
4. Mechanical Arrangement

Principle:

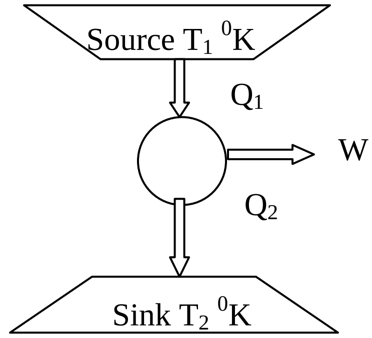


Fig.6

Hence, $Q_1 = W + Q_2$

Here, $Q_1 =$ Heat taken from the Source

$Q_2 =$ Heat given to the Sink

$W =$ Work done

Therefore, the efficiency of heat engine $= \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

The efficiency of Carnot engine lies between $\mathbf{0 < \eta \leq 1}$**.**

2.2 The Refrigerator:

The efficiency of a refrigerator is expressed in terms of the coefficient of performance (COP), which we denoted with the symbol β. For a refrigerator the objective, that is, the energy sought, is Q_1 the heat transferred from the refrigerated space. The energy that costs is the work, W. Thus, the COP, β, is

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1 - \eta}{\eta}$$

The value of COP, β, lies between the $\mathbf{2 < \beta \leq 6}$**.**

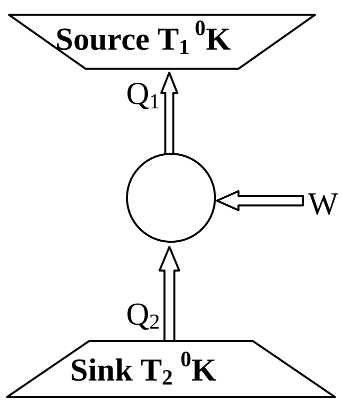


Fig.7

3. Reversible Process and Irreversible Process:

If heat is absorbed by the substance in the direct process, the same quantity will be given out by it in the reverse process, and if work is done by the substance in the direct process, an equal amount of work done will be done on the substance in the reverse process. Thus there is no wastage of energy at all in the reversible process.

Those processes which cannot be reacted in the opposite order by reversing the controlling factors are known as an irreversible process.

4. Carnot's Engine and Carnot's Cycle: A heat engine is a practical setup to convert heat into the mechanical work. The French engineer, Nicolas Leonard Sadi Carnot, who expressed the foundations of the second law of thermodynamics in 1824 and conceived an ideal engine free from all the imperfectness of actual engines. Hence, there is no ideal engine in actual practice. This imaginary engine is, however, taken as a standard against which the performance of actual engines is checked.

Structure: This has the four parts.

(a) A cylinder with perfectly non-conducting walls and perfectly conducting base containing the working substance and fitted with frictionless and perfectly insulating piston upon which weight can be placed.

(b) A hot body of large heat capacity having the high temperature T_1 ^{0}K works as ***Source of Heat.***

(c) A cold body of large heat capacity having the temperature T_2 ^{0}K $(T_1 > T_2)$works as ***Sink.***

(d) A perfectly insulating platform works as the ***Stand*** for the cylinder.

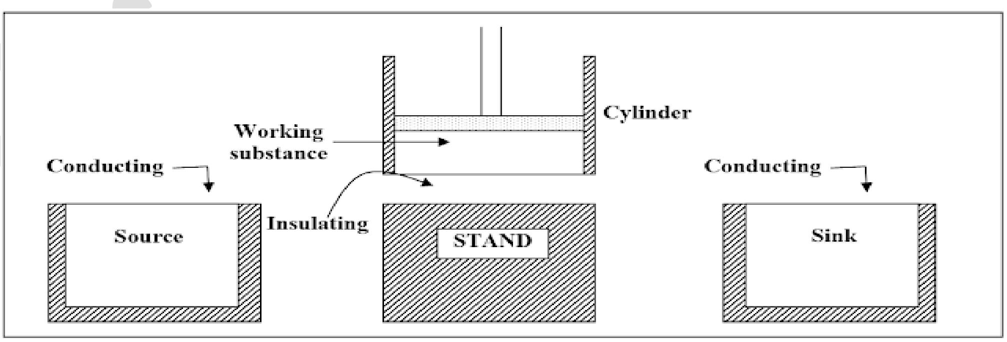


Fig. 8

The cylinder can be placed on any one of the Source, Sink and Stand and there no work is done for placing the cylinder from one to another.

Carnot's Cycle: The working substance is complete one cycle in four processes, consisting of two isothermal processes and two adiabatic processes. This cycle is known as Carnot's Cycle. This cycle has been representing on P-V diagram and shown in **fig.9.**

Let us consider the quantity of one gram mole of the working substance in the cylinder that has been taken and the pressure P_1 and the volume V_1 have been shown at point A on the P-V diagram.

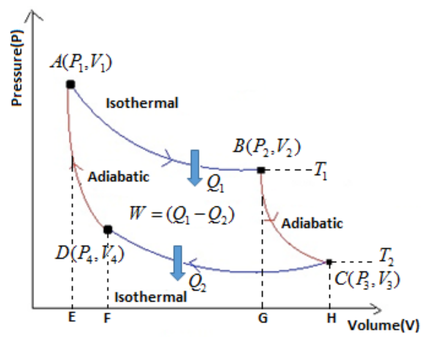


Fig.9: Carnot's Cycle on P-V diagram.

(a) Operation I: Firstly the cylinder is placed on the Hot Source. The pressure is reduced slowly-slowly from the piston. The working substance thus expands doing external work raising the piston. The working substance thus isothermally expends and the substance has been taken the heat from the source maintain the constant temperature T_1. Suppose for maintaining the temperature T_1, the heat Q_1 has taken from the source. This operation is represented by the isothermal curve AB on the P-V diagram. The Pressure P_2 and the volume V_2 is represented at point B. Thus

Work done in the process AB = $W_1 = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{RT_1}{V} dV$ $\quad\quad \therefore PV = RT_1 \Rightarrow P = \frac{RT_1}{V}$

$$W_1 = RT_1 \log_e \left(\frac{V_2}{V_1}\right)$$

Absorb heat from the source = $Q_1 = W_1 = RT_1 \log_e \left(\frac{V_2}{V_1}\right) = Area\ ABGEA$ --------- (1)

(b) Operation II: Now the cylinder is removed from the Hot Source and placed at non-conducting the stand. The weight is further reduced from the piston; the substance is allowed to expand. This expansion is adiabatic because no heat can enter or leave the system. The system performs the work W_2 in raising the piston at the expense of its internal energy and its temperature therefore falls to T_2 ^{0}K. This operation is represented by the adiabatic curve BC on the P-V diagram. The Pressure P_3 and the volume V_3 is represented at point C. Thus

Work done in the process BC = $W_2 = \int_{V_2}^{V_3} PdV = \int_{V_2}^{V_3} \frac{K}{V^\gamma} dV$ $\quad\quad \therefore PV^\gamma = K \Rightarrow P = \frac{K}{V^\gamma}$

$$W_2 = \frac{K}{1-\gamma}\left(V_3^{1-\gamma} - V_2^{1-\gamma}\right) \quad\quad \therefore P_2V_2^\gamma = P_3V_3^\gamma = K$$

$$W_2 = \frac{1}{1-\gamma}(P_3V_3 - P_2V_2)$$

$$W_2 = \frac{R}{1-\gamma}(T_2 - T_1) \quad\quad \therefore P_2V_2 = RT_1 \& P_3V_3 = RT_2$$

$$W_2 = \frac{R}{1-\gamma}(T_2 - T_1) = Area\ BCHGB \quad\quad \text{--------- (2)}$$

(c) Operation III: Now cylinder is removed from the Stand and placed at the Sink ($T_2\ {}^0K$). The weight is increasing slowly on the piston. The working substance is thus compressed until its pressure and volume becomes P_4, V_4 represented by the point D. The working substance thus isothermally compresses and the substance has been rejected the heat Q_2 to the Sink maintain the constant temperature T_2. Suppose for maintaining the temperature T_2, the heat Q_2 has rejected to the sink. This operation is represented by the isothermal curve CD on the P-V diagram. Thus

Work done in the process CD = $W_2 = \int_{V_3}^{V_4} PdV = \int_{V_3}^{V_4} \frac{RT_2}{V} dV$ $\quad\quad \therefore PV = RT_2 \Rightarrow P = \frac{RT_2}{V}$

$$W_3 = RT_2 \log_e \left(\frac{V_4}{V_3}\right)$$

Heat reject to the sink = $Q_2 = W_3 = RT_2 \log_e \left(\frac{V_4}{V_3}\right) = Area\ CHFDC$ --------- (3)

(d) Operation IV: Now the cylinder is removed from the Sink and further placed at non-conducting the stand. The weight is further slightly increased on the piston so that the substance undergoes a slow adiabatic compression and its temperature rises. This compression is continued until the temperature rises to T_1 and the substance come back to its original position i.e. the Pressure P_1 and the volume V_1 and the gas goes to be ready again to work. This operation is represented by the adiabatic curve DA on the P-V diagram. Thus

Work done in the process DA = $W_4 = \int_{V_4}^{V_1} PdV = \int_{V_4}^{V_1} \frac{K}{V^\gamma} dV$ $\quad\quad \therefore PV^\gamma = K \Rightarrow P = \frac{K}{V^\gamma}$

$$W_4 = \frac{K}{1-\gamma}\left(V_1^{1-\gamma} - V_4^{1-\gamma}\right) \quad\quad \therefore P_1V_1^\gamma = P_4V_4^\gamma = K$$

$$W_4 = \frac{1}{1-\gamma}(P_1V_1 - P_4V_4)$$

$$W_4 = \frac{R}{1-\gamma}(T_1 - T_2) \quad\quad \therefore P_1V_1 = RT_1 \& P_4V_4 = RT_2$$

$$W_4 = \frac{R}{1-\gamma}(T_1 - T_2) = Area\ ADFEA \quad\quad \text{--------- (4)}$$

The work done by the engine per cycle:

Net amount of work done by the engine per cycle = $W = W_1 + W_2 + W_3 + W_4$

$$W = RT_1 \log_e\left(\frac{V_2}{V_1}\right) + \frac{R}{1-\gamma}(T_2 - T_1) + RT_2 \log_e\left(\frac{V_4}{V_3}\right) + \frac{R}{1-\gamma}(T_1 - T_2)$$

$$W = RT_1 \log_e\left(\frac{V_2}{V_1}\right) + RT_2 \log_e\left(\frac{V_4}{V_3}\right) \qquad \text{--------- (5)}$$

The points B and C are at the same adiabatic curve BC, so that

$$T_1 {V_2}^{\gamma-1} = T_2 {V_3}^{\gamma-1}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{{V_3}^{\gamma-1}}{{V_2}^{\gamma-1}} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \qquad \text{--------- (6)}$$

And points D and A are at the same adiabatic curve DA, so that

$$T_2 {V_4}^{\gamma-1} = T_1 {V_1}^{\gamma-1}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{{V_4}^{\gamma-1}}{{V_1}^{\gamma-1}} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \qquad \text{--------- (7)}$$

From eq. (6) and eq. (7), we get

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \text{--------- (8)}$$

Thus from eq. (5) and eq. (8), we obtain

$$W = RT_1 \log_e\left(\frac{V_2}{V_1}\right) + RT_2 \log_e\left(\frac{V_1}{V_2}\right) \qquad \text{--------- (9)}$$

$$\Rightarrow W = R(T_1 - T_2) \log_e\left(\frac{V_2}{V_1}\right) \qquad \text{--------- (10)}$$

From the First Law of Thermodynamic,

$$dQ = dU + W$$

$\because dU = 0$ Because of the working substance is obtained to its initial position. Therefore

$$dQ = W = R(T_1 - T_2) \log_e\left(\frac{V_2}{V_1}\right)$$

$$Q_1 - Q_2 = R(T_1 - T_2) \log_e\left(\frac{V_2}{V_1}\right) \qquad \text{--------- (11)}$$

Efficiency of the Carnot's Cycle: It is defined as

The efficiency of Carnot's engine, $\eta = \frac{W}{Q_1} = \frac{Heat\ coverted\ into\ work}{Heat\ taken\ from\ the\ source} = \frac{Q_1 - Q_2}{Q_1}$

$$\eta = \frac{R(T_1 - T_2)\log_e\left(\frac{V_2}{V_1}\right)}{RT_1 \log_e\left(\frac{V_2}{V_1}\right)} = \frac{T_1 - T_2}{T_1}$$

$$\Rightarrow \eta = 1 - \frac{T_2}{T_1} \qquad \text{--------- (12)}$$

It is clear from the eq. (12), the efficiency of the engine depends upon the temperatures T_1 and T_2 of the source and Sink, and greater the difference between T_1 and T_2, the greater is the efficiency, and not depend upon the working substance. Because it is impossible; the temperature of the sink cannot be absolute zero i.e. $T_2 \neq 0$ so $\eta \neq 0$. Therefore the efficiency of Carnot's engine cannot be 100 %. So an engine with 100% efficiency is not practically possible.

From eq. (8),

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} = \rho\ (Say)$$

Where ρ is the adiabatic expansion ratio.

Therefore, from eq. (6)

$$\frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} = \rho^{\gamma-1}$$

Thus from eq. (12)

$$\eta = 1 - \frac{1}{\rho^{\gamma-1}} = 1 - \rho^{1-\gamma} \qquad \text{--------- (13)}$$

Reversibility of Carnot's Engine:

The Carnot's engine is perfectly reversible because all changes are being with the slow process. There is no friction between the cylinder and the piston. There is no loss of any part of heat due because the piston and cylinder are perfectly insulating. Therefore, Carnot's engine is a perfectly reversible engine.

5. Carnot's Theorem:

Statement: This theorem consists of two parts and may be defined as follows:

(1) ***No engine working between two given temperatures can be more efficient than Carnot's engine (reversible engine) working between the same two temperatures and***

(2) ***All reversible engine working between the same two temperatures have same efficiency, whatever be the working substance.***

Proof of first Part: Suppose there are two heat engines A and B operating between the temperatures T_1 and T_2 $(T_1 > T_2)$. Let engine A be the reversible and B be the irreversible and shown in fig. 2 (a). Let the engine A absorbs the heat Q_1 from the source, performs the work W and the heat reject to the sink is $Q_2 = (Q_1 - W)$. The efficiency of this engine A will be, $\eta_A = W/Q_1$.

Similarly, let the engine B takes Q'_1 heat from the source, perform work W and reject the heat $Q'_2 = (Q'_1 - W)$ to the sink. The efficiency of this engine A will be, $\eta_B = W/Q'_1$.

Let us assume that the efficiency of irreversible engine B is greater than that of the reversible engine A. i.e.

$$\eta_B > \eta_A$$

or $\frac{W}{Q'_1} > \frac{W}{Q_1}$

or $Q'_1 < Q_1$

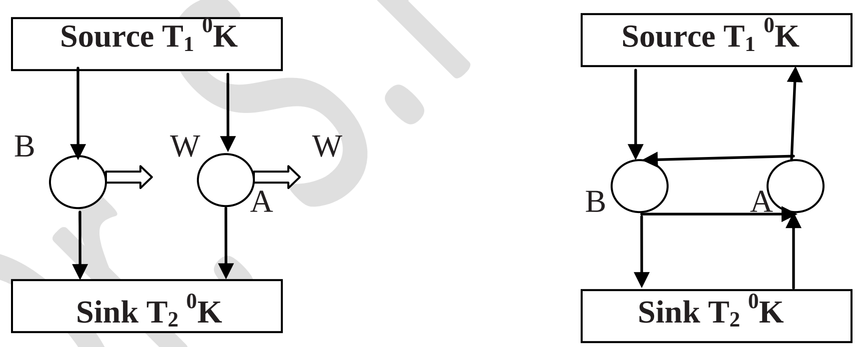
Thus, $Q_1 - Q'_1 > 0$ i.e. $Q_1 - Q'_1 = +ve$ quantity.

Now, suppose that the two engines A and B are coupled together by a belt in such a way that B works forward and A in reverse direction as shown in fig. 2 (b). The engine A now works as a refrigerator driven by engine B, taking the heat $Q_1 - W$ from the sink. The work W is done on it and heat $Q_1$is given out to the source. The work W required to be done on H is directly supplied by B, working forward. In this way the engine B and A are coupled together to a belt from a self acting machine. For the complete system as shown in fig. 2 (b), in a complete cycle, the net heat taken in from the sink will be,

$$= Heat\ taken\ in\ by\ A - Heat\ reject\ by\ B$$

$$= (Q_1 - W) - \left(Q'_1 - W\right) = \left(Q_1 - Q'_1\right)$$

Thus the coupled device is transferring an amount of heat $Q_1 - Q'_1$in each cycle from the sink to the source without the aid of any external energy. But this transfer of form cold body to hot body without any external work is contradictory to the second law of thermodynamics and it is impossible. Therefore our assumption is wrong that $\eta_B > \eta_A$.i.e. a reversible engine is more efficient than an irreversible engine.



Proof of Second Part: Suppose engine A and B are reversible engines, working between the same source and sink. If we consider that A derives B backward, then A cannot be more efficient than B. Similarly, if we suppose that B derives A backward, then B cannot be more efficient than A. Hence A and B are equally efficient. Thus, the efficiency of all reversible engines, working between the same two temperatures, is the same irrespective of the nature and properties of working substance. In other words the efficiency of a reversible engine depends upon the temperatures of the source and sink and is independent of the nature of the working substance.

6. <u>Thermodynamical Scale of temperature</u> or <u>Absolute Scale of temperature</u> or <u>Kelvin Scale of temperature:</u>

The Lord Kelvin defined the a temperature scale which is independent of the properties of any particular substance and called an Absolute Scale of temperature or Kelvin Scale of temperature or Thermodynamical Scale of temperature.

According to the Leonard Sadi Carnot, *the efficiency of a reversible engine depends only upon the two temperatures between which it is working and is independent of working substance.*

In 1848, taking the advantage of its Leonard Kelvin made a hypothetical temperature scale, which is independent of the properties of matter and known as the Absolute Scale of temperature or Kelvin Scale of temperature or Thermodynamical Scale of temperature.

Let us suppose a reversible engine takes in a quantity of heat Q_1 at the temperature θ_1 and rejects the heat Q_2 at the temperature θ_2, then according to Carnot' engine, therefore, the efficiency of the engine is given by $\eta = f(\theta_1, \theta_2)$

Where $f(\theta_1, \theta_2)$ is any definite function, So that

$$\eta = 1 - \frac{Q_2}{Q_1} = f(\theta_1, \theta_2)$$

$$\frac{Q_1}{Q_2} = \frac{1}{1 - f(\theta_1, \theta_2)}$$

$$\frac{Q_1}{Q_2} = F(\theta_1, \theta_2) \quad \text{------ (1)}$$

Where $F(\theta_1, \theta_2)$ represents another function of θ_1 and θ_2.

Now suppose that a reversible engine takes in a quantity of heat Q_2 at the temperature θ_2 and rejects the heat Q_3 at the temperature θ_3, $(\theta_2 > \theta_3)$. Therefore,

$$\frac{Q_2}{Q_3} = F(\theta_2, \theta_3) \quad \text{------ (2)}$$

If reversible engine works as such takes in a quantity of heat Q_1 at the temperature θ_1 and rejects the heat Q_3 at the temperature θ_3, $(\theta_1 > \theta_3)$. Therefore,

$$\frac{Q_1}{Q_3} = F(\theta_1, \theta_3) \quad \text{------ (3)}$$

By multiplying eq. (1) and eq. (2), then

$$\frac{Q_1}{Q_2}\frac{Q_2}{Q_3} = F(\theta_1, \theta_2)F(\theta_2, \theta_3)$$

$$\frac{Q_1}{Q_3} = F(\theta_1, \theta_2)F(\theta_2, \theta_3) \quad \text{------ (4)}$$

From eq. (3) and eq. (4), we have

$$F(\theta_1,\theta_3) = F(\theta_1,\theta_2)F(\theta_2,\theta_3) \qquad \text{------ (5)}$$

This eq. (5) is called a Functional Equation. There on θ_2, on the left-hand side, So, the right-hand side of eq. (5) should not contain θ_2. It is possible if

$$F(\theta_1,\theta_2) = \frac{\Phi(\theta_1)}{\Phi(\theta_2)} \;\&\; F(\theta_2,\theta_3) = \frac{\Phi(\theta_2)}{\Phi(\theta_3)}$$

Where Φ is another function of θ. These values putting in functional eq. (5), we have

$$F(\theta_1,\theta_3) = \frac{\Phi(\theta_1)}{\Phi(\theta_2)}\frac{\Phi(\theta_2)}{\Phi(\theta_3)}$$

$$F(\theta_1,\theta_3) = \frac{\Phi(\theta_1)}{\Phi(\theta_3)} \qquad \text{------ (6)}$$

From eq. (1),

$$\frac{Q_1}{Q_2} = F(\theta_1,\theta_2) = \frac{\Phi(\theta_1)}{\Phi(\theta_2)} \qquad \text{------ (7)}$$

But $\theta_1 > \theta_2$ and $Q_1 > Q_2$, therefore from the eq.(7)

$$\Phi(\theta_1) > \Phi(\theta_2)$$

It is clear that $\Phi(\theta)$ increases as increase of temperature. Therefore it can be used for temperature measurement. If $\Phi(\theta)$ represents as τ for the new scale of temperature, so from eq. (7),

$$\frac{Q_1}{Q_2} = \frac{\Phi(\theta_1)}{\Phi(\theta_2)} = \frac{\tau_1}{\tau_2} \qquad \text{------ (8)}$$

This eq. (8) defines ***the Thermodynamical scale of temperature or Kelvin's absolute thermodynamic scale of temperature***.

The ratio of any two temperatures on this scale is equal to the ratio of quantities of heat taken in and reject by a Carnot reversible engine working between these temperatures. It is independent of the nature of any particular substance. From eq. (8)

$$\frac{Q_1}{Q_2} = \frac{\tau_1}{\tau_2} \qquad \text{------ (9)}$$

$$\frac{Q_2}{Q_1} = \frac{\tau_2}{\tau_1}$$

$$1 - \frac{Q_2}{Q_1} = 1 - \frac{\tau_2}{\tau_1}$$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{\tau_2}{\tau_1} \qquad \text{------ (10)}$$

Work done by the engine par cycle (W)

$$W = Q_1 - Q_2 = \tau_1 - \tau_2 \qquad \text{------ (11)}$$

Therefore, this is also called ***the Kelvin Work or Work scale of temperature***.

I. **Negative Temperature is not possible on the Absolute Scale of Temperature.**

From eq. (10),

$$\eta = 1 - \frac{\tau_2}{\tau_1}$$

If the measure of negative temperature is possible, $\tau_2 = -k$, then

$$\eta = 1 - \frac{(-k)}{\tau_1} = 1 + \frac{k}{\tau_1}$$

i.e. $\eta > 1$ (Which impossible). Therefore, it is impossible to measure the negative temperature on the Absolute Scale of Temperature.

II. **Zero of the Absolute Scale of Temperature.**

From eq. (10)

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{\tau_1 - \tau_2}{\tau_1}$$

The value of η equal to one when $\tau_2 = 0$ and $Q_2 = 0$. Thus the zero of absolute scale is that temperature of the sink at which no heat is rejected to it, the efficiency of the engine being unity i.e. all the heat taken by the engine is converted into work.

III. **Size of the degree of absolute scale of the temperature.**

On thermodynamical scale, the efficiency of Carnot's engine is given as

$$\eta = 1 - \frac{\tau_2}{\tau_1}$$

On the perfect gas scale, the efficiency of Carnot's engine is given as

$$\eta = 1 - \frac{T_2}{T_1}$$

Let us consider the Carnot's engine is working between ice point and steam point. It has been shown on PV-Diagram fig. 1, therefore,

$$\eta = 1 - \frac{\tau_{ice}}{\tau_{steam}} = \frac{\tau_{steam} - \tau_{ice}}{\tau_{steam}} = \frac{W}{Q_1}$$

$$\text{W} = \text{Work done by the per cycle} = \tau_{steam} - \tau_{ice} = Area\ ABCD$$

Let this area be divided into 100 equal parts by drawing isothermals parallel to CD or AB. Then any isothermal will be 1^0 lower than the isothermal just above it and the area of each equal part will correspond to one degree on the absolute scale.

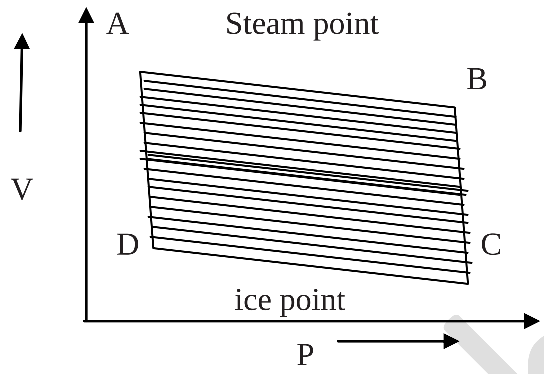


Fig. 11

IV. **Identity of absolute scale of temperature with perfect gas scale of temperature:**

a. On thermodynamical scale, the efficiency of Carnot's engine is given as

$$\eta = 1 - \frac{\tau_2}{\tau_1}$$

On the perfect gas scale, the efficiency of Carnot's engine is given as

$$\eta = 1 - \frac{T_2}{T_1}$$

Therefore, $\frac{\tau_2}{\tau_1} = \frac{T_2}{T_1}$ ------ (12)

i.e. the ratio of any two temperature on the thermodynamic scale is the same as on the perfect gas scale.

b. On perfect gas scale of temperature

$$T_{Steam} - T_{ice} = 100$$

On absolute scale of temperature,

$$\tau_{Steam} - \tau_{ice} = 100$$

So,

$$\tau_{Steam} - \tau_{ice} = T_{Steam} - T_{ice}$$

$$\eta = 1 - \frac{\tau_{ice}}{\tau_{Steam}} = 1 - \frac{T_{ice}}{T_{Steam}}$$

$$T_{Steam} = \tau_{Steam}$$

Therefore, steam points are identical on both temperature scales.

c.

$$\tau_{Steam} - \tau_{ice} = T_{Steam} - T_{ice}$$

$$T_{ice} = \tau_{ice}$$

i.e. Therefore, ice points are identical on both temperature scales.

d.

$$\frac{\tau_2}{\tau_1} = \frac{T_2}{T_1}$$

let the temperature of sink at ice point, then

$$\frac{\tau_{ice}}{\tau_1} = \frac{T_{ice}}{T_1}$$

$$T_1 = \tau_1$$

Therefore, general temperature points are identical on both temperature scales.

e.

$$\frac{\tau_2}{\tau_1} = \frac{T_2}{T_1}$$

if $T_2 = 0$, we must also have $\tau_2 = 0$ so that the zero of the absolute scale is identical with the zero of the perfect gas scale. Therefore, both scale of temperature are completely identical in all form.